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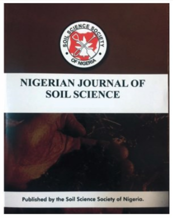

# Evaluation of furrow infiltration by Swartzendruber and Horton's models in Northern Guinea savanna of Nigeria

**H.G. Aminu[1], O.J. Mudiare[1], M.A. Oyebode[1], *A. Abdulkadir[2] and Z. Aminu[3]**

*[1]Department of Agricultural and Bio-Resources Engineering, ABU, Zaria, Nigeria.*
*[2]Department of Soil Science, ABU, Zaria, Nigeria.*
*[3]Department of Geography, FUBK, Birnin Kebbi, Nigeria.*

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Corresponding Author's E-mail Address:
akad.aisha@gmail.com ; +2348023619819

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**ABSTRACT**

The study aimed at evaluating two models (Swartzendruber and Horton) using furrow infiltration data measured in Samaru, Zaria. These measurements were carried out on the three field plots A, B and C. Infiltration parameters were generated from the data measured from plot A and B and fitted into the models for predictions of water infiltrated depths at different time intervals. The predictions from the models were compared with the measured field data from plot C. Statistical indices such as coefficient of determination ($R^2$), root mean square error (RMSE) and T-test at 5% level of significance were used to determine the best performing model. The results show that, value of $R^2$ and RMSE recorded for Horton's model was 0.996 and 3.05, while the value of $R^2$ and RMSE recorded for Swartzendruber's model was 0.998 and 3.01, respectively. The T-test values obtained for Horton's model was 2.93, while 2.51 was recorded for Swartzendruber's model. The results of the evaluation indicated that both models were considered suitable because they presented high values of $R^2$ and low values of RMSE as suggested by Aminu (2019). Similarly, the results obtained from the T-test statistics indicated that there is a no significant difference between the models evaluated because the calculated values of 'T' 2.93 and 2.51were greater than the tabulated value 2.365 at 5% confidence interval. Therefore, this study recommends that both models should be used to predict infiltration rates of soils in the study area.

## 1.0 Introduction

The theory of infiltration is based on the analysis of soil water movement under unsaturated conditions (Eldeiry *et al*., 2005). There are many factors that can influence the infiltration behaviour of soils and these include: initial soil moisture content, soil type, soil hydraulic conductivity, and surface properties (such as slope, vegetation cover and plant roots). Infiltration has been investigated by authors such as (Kostiakov, 1932; Horton, 1940; and Swartzendruber, 1987). Infiltration models have also been revised by authors such as Singh (2007) and Parhi (2007) amongst others. Similarly, contributions were also made on them by other set of authors such as Amir *et al.* (2011), Salman *et al*. (2013), Lentz and Bjorneberg (2014). Infiltration plays an important role in surface and subsurface hydrology, soil erosion, runoff generation, design of irrigation systems, management of irrigation systems, and simulating models for performance prediction or optimization for the whole field (Salman *et al.*, 2013). As it has been noted that Horton in the year (1933) first conceived the theory of infiltration capacity which was generally applicable to rainfall intensity graph but later refined the word capacity in the year (1940) by referring it to infiltration rates which declines exponentially during a given storm. The parameters of the equation developed were only obtained with simulated rainfall data. However, the Swartzendruber (1987) equation provided a series solution that holds for small, intermediate and large time (infiltration times) and when differentiated, it takes the form of the Horton's (1940) equation which could be useful for both simulated rainfall data and furrow infiltration data. The aim of the study is to evaluate the performance of these models (Swartzendruber and Horton) using measured field data to and compare their predicting performance.

## 2.0. Materials and methods

### *2.1. Study Area*

The measurements were carried out at the experimental field located in Samaru College of Agriculture, Division of Agricultural Colleges, Ahmadu Bello University, Zaria, which is located in Zaria lies within latitude 11°11'N and longitude 7° 38'E, and at an altitude of about 686m above mean sea level. The precipitation in this region is observed within the months of May and September with an average annual long-term value (50 years) of 1016mm (Yamusa and Abdulkadir, 2020). Average minimum and maximum temperatures (1968-2017) are 13.3˚C and 29.7˚C (Yamusa and Abdulkadir, 2020). The soils in the area varied between loam, sandy loam and clay loam and have values for bulk densities ranging between 1.25 g/cm$^3$ – 1.80g/cm$^3$.

### *2.2. Experimental Procedure*

The field layout that was used for this research work consisted of three plots A, B and C of six furrows each. Footpaths of 1m were also provided after every 6th furrow at the center line between each plot and a waste furrow of 0.5m for excess discharge. Three furrows were chosen at a time for each method used for data collection, the main objective was to ensure that measurements could easily be carried out in the furrows in-between to prevent lateral flow and also monitoring of irrigation events on the site. The total area of the field was 45m by 16.25 m, while the width for each plot within the field was 4.5m. The length and width of each furrow selected were 40m and 0.75m, respectively, in order to prevent longer advance times of flow towards the end of the field. Measuring stations were established at every 5m along the furrow length where successive measurements of discharge rates were recorded. Water delivered into the field channel was diverted into the furrows and cutthroat flumes were placed at the upper and lower ends of the furrows to measure the inflow and outflow rates. The inflow '$Q_i$' into the furrow was measured by a cut throat flume placed at 2m from the furrow inlet as water is conveyed into the furrow and the corresponding outflow '$Q_o$' was also measured as the wetted front advanced to the end of the field by an outflow flume. While the inflow was assumed constant, the outflow was measured at successive time intervals. The area of the furrow was obtained by multiplying the furrow length and top width of the furrow. The infiltrated volume was therefore obtained by subtracting the outflow from the inflow and dividing the result by the measured area of furrow. The inflow flume was placed 2m from the field inlet for measuring depth of flow. The infiltration rate is calculated as follows by Merriam and Keller (1978):

$$f = \frac{Qi - Qo}{LT} \qquad \text{Eqn.1}$$

where:

f = the infiltration rate at time t in (m/min)

$Q_i$ = inflow discharge (m$^3$ min$^{-1}$)

$Q_o$ = outflow discharge (m$^3$ min$^{-1}$)

L = length of the furrow (m)

T = top width at the surface (m).

The form of equation for the inflow rate obtained from the field calibration is given as:

$$Q = 0.060H^{1.089} \qquad \text{Eqn.2}$$

where:

H = depth of water flow in the flume

The values 0.060 and 1.089 were the intercept and slope obtained from the graph.

### *2.3 Theoretical framework*

*Infiltration Models*

Over the years, several infiltration models have been developed and used in surface irrigation modelling owing to their simplicity and minimal data requirement. In this research, the models studied include; Swartzendruber model (SW) and the Horton's model. The Swartzendruber's model will be used as a reference model to compare the Horton's model for validation. A brief description of the infiltration models used in this study includes:

*Swartzendruber's Model*

The Swartzendruber (1987) model provides a series solution that holds for small, intermediate and large time (infiltration times) and allows for surface ponding. The equation as describe by Swartzendruber (1987) is given as:

$$I = f_c t + \frac{c}{d}[1 - \exp(-dt^{0.5})] \qquad \text{Eqn. 3}$$

where:

I = cumulative infiltration (mm)

$f_c$ = steady state infiltration rate (mm/hr)

t = infiltration time (hr)

c and d are empirical coefficients

***Horton's Model***

Horton (1940) presented a three-parameter infiltration equation which may be written as:

$$I = f_c + (f_o - f_c)e^{-kt} \qquad \text{Eqn.4}$$

Where:

I = infiltration rate (mm/hr)

$f_c$ = steady state infiltration rate (mm/hr)

$f_o$ = infiltration rate at t = 0 (mm/hr)

t = infiltration time (hr)

However, Mirzaee *et al.* (2014) reviewed the Horton's model and obtained an equation with the form:

$$I = ct + m(1 - e^{-kt}) \qquad \text{Eqn.5}$$

where:

c, m and k = empirical coefficients determined from observed infiltration data. Other terms as previously defined.

### *2.4. Model Parameter Estimation*

The parameters $f_o$ and k must depend on the initial water content as well as application rate; and for homogeneous profiles, $f_c$ will be somewhat smaller than the saturated hydraulic conductivity.

Swartzendruber's equation is simplified and presented in Eqn. 6 as:

$$I = f_c t + \quad (1\,(1 - e^{-d\sqrt{t}}\,) \qquad \text{Eqn.6}$$

The integral form of Horton's model in Eqn. 2 can also be given as:

$$F(t) = \int f_c dt + (f_o - f_c) \int_0^t e^{-kt}\, dt \qquad \text{Eqn.7}$$

The simplified form of Eqn. 7 is presented in Eqn. 8. However, it is similar to Swartzendruber's (1987) equation except for the fact that exponent of infiltration time has no square root.

$$I = f_c t + \frac{fo - fc}{k}(1 - e^{-kt}) \qquad \text{Eqn.8}$$

where:

$f_c \equiv c$ $\qquad \frac{c}{d}$

$\frac{fo-fc}{k} \equiv m$

The value $f_o$ and $f_c$ can be determined by plotting the values of infiltration rate versus time (Abdulkadir *et al.*, 2011).

The infiltration rate f is determined by differentiating Eqn. 1 with respect to time (t) to give:

$$f - f_c = \frac{c}{2\sqrt{t}} e^{-d\sqrt{t}} \qquad \text{Eqn.9}$$

Eqn. 9 is rearranged to give

$$(f - f_c)\sqrt{t} = \frac{c}{2} e^{-d\sqrt{t}} \qquad \text{Eqn.10}$$

The values of c and d can be obtained by using least square regression techniques. However, the complete use of least square approach gives the predicted value of I values much higher than the measured. Thus, c values can be determined by optimization technique once the value of d is obtained. Similarly, the parameters for the k and m can be obtained by least square approach. The bulk density and available moisture holding characteristics as reported by Lentz and Bjorneberg (2014) is given as:

$$\rho = \frac{Ws}{v} \qquad \text{Eqn.11}$$

where:

$\rho =$ bulk density (g/cm$^3$)

Ws = weight of oven dry soil (g)

v = bulk volume of soil (cm$^3$)

## 3.0 Results and discussion

### *3.1. Soil Properties*

Soil samples taken from the field were used to determine soil properties such as particle size distribution and textural classes and bulk density. The summary of the results from the laboratory are presented in Table 1

Table 1: Soil physical properties of the experimental site

| **Depth (m)** | **Particle Size (%)** | | | **Texture** | **Bulk Density** |
|---|---|---|---|---|---|
| | **Clay** | **Silt** | **Sand** | | **(g/cm$^3$)** |
| 0.0 – 0.2 | 18 | 31 | 51 | L | 1.25 |
| 0.2 – 0.4 | 14 | 32 | 59 | SL | 1.37 |
| 0.4 – 0.6 | 18 | 37 | 45 | L | 1.55 |
| 0.6 – 0.8 | 28 | 41 | 31 | CL | 1.80 |

L= Loam; SL=sandy loam; CL=clay loam

The soils in the area varied between loam, sandy loam and clay loam and have values for bulk densities ranging between 1.25 and 1.80g/cm$^3$. Similarly, the organic matter content determined also varied between 1.41, 1.62 and 1.77%. More so, the moisture holding characteristics for the experimental site varied between 71mm, 73mm and 77mm, respectively.

### *3.2. Measured Infiltrated Depths*

The accumulated infiltrated depths were measured at the experimental site using the inflow-outflow method. Table 2 shows the accumulated infiltrated depths measure during the first irrigation trials.

Table 2: Soil infiltration data measured in plot A during the first irrigation event

| **Parameters** | **Time of Reduced Inflow (min)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **0** | **5** | **10** | **15** | **20** | **30** | **40** | **50** | **60** |
| H (m) | - | 0.040 | 0.046 | 0.049 | 0.052 | 0.055 | 0.059 | 0.060 | 0.068 |
| $Q_{in}$ (m$^3$/min) | 0.230 | 0.230 | 0.230 | 0.230 | 0.230 | 0.230 | 0.230 | 0.230 | 0.230 |
| $Q_{out}$ (m$^3$/min) | - | 0.108 | 0.125 | 0.134 | 0.143 | 0.152 | 0.165 | 0.168 | 0.170 |
| f x$10^{-3}$ (m/min) | - | 4.067 | 3.50 | 3.24 | 2.90 | 2.60 | 2.17 | 2.07 | 1.83 |
| I (mm) | - | 20 | 35 | 48 | 58 | 78 | 86 | 103 | 110 |

H = Depth of flow in the flume; Qin = Inflow discharge; Qout = Outflow discharge; f = infiltration rate; I = Accumulated infiltrated depth.

The water infiltrated depths were measured in the furrows of the experimental site for plot A, B and C from 5 minutes to 60 minutes, and inflow time was cut off at 65 minutes. 20mm and 110mm depth of water was measured and recorded during the first irrigation event, 26mm and 57mm was measured during the second irrigation event, while 28mm and 106mm was recorded for the third irrigation event. Similarly, 29mm and 98mm were recorded for plot "B" during the first irrigation events, 25mm and 90mm for the second irrigation, and 25mm and 90mm for the third irrigation events. The minimum and maximum infiltrated depths recorded for plot "C" during the first irrigation event was 23mm and 65mm, while 25mm and 63mm were recorded during the second irrigation event, and finally 27mm and 72mm recorded for the third irrigation event. The variation of accumulated infiltrated depths observed within the three plots was due to spatial variability of the soil in the study area, flow rates and slope of the field as also reported by (Aminu *et al*., 2019).

Table 3: Estimated parameters for Swartzendruber's and Horton's models

| **Parameters** | **Swartzendruber's model** | | **Horton's model** | |
|---|---|---|---|---|
| | PLOT A | PLOT B | PLOT A | PLOT B |
| c | 91 | 93 | 91 | 93 |
| k | 462 | 333 | 0.31 | 0.30 |
| m | 2.83 | 2.08 | 1.12 | 1.06 |
| r | 0.848 | 0.845 | 0.984 | 0.932 |

The parameters for Swartzendruber's model were obtained from optimization techniques and least square approach while the parameters for Horton's model were obtained by optimization techniques as presented in Table 3. Regression coefficients (r) were obtained from the graphs of cumulative infiltration plotted against time. The parameters for Swartzendruber's model have values of regression coefficients 0.848 recorded in plot A and 0.845 recorded in plot B. Similarly, the parameters for Horton's model have values of regression coefficients 0.984 and 0.932 in plot A and B, respectively. The parameters for both models performed well because they presented least square regression values as high as 0.984 for Horton's model and 0.848 for Swartzendruber's model. However, the idea of having an infiltration model with a greater number of fitting parameters and greater magnitude of least square regression value is always considered to perform best in predicting soil infiltration rates as reported by Parhi *et al.* (2007).

The predicted values for these models were used to plot graphs of cumulative infiltration 'F' (mm) against time 't' in Figure 1 and 2. The plotted data for both models were observed to be well fitted on the line in Fig. 2. This indicated that there was a good agreement between the predicted cumulative infiltrated depths for the two models at different time interval. However, the trend from the two graphs indicated that the predicted values for both Horton and Swartzendruber's model were well fitted as compared to the predictions in plot A. Thus; the result from the evaluation indicated that both models will be suitable for predicting infiltration rates of soil using measured field data. However, the variability encountered could be due to factors such as soil conditions, season and other hydrological phenomena that Horton (1933) noted as limitations of the model when he first conceived the theory of infiltration capacity.

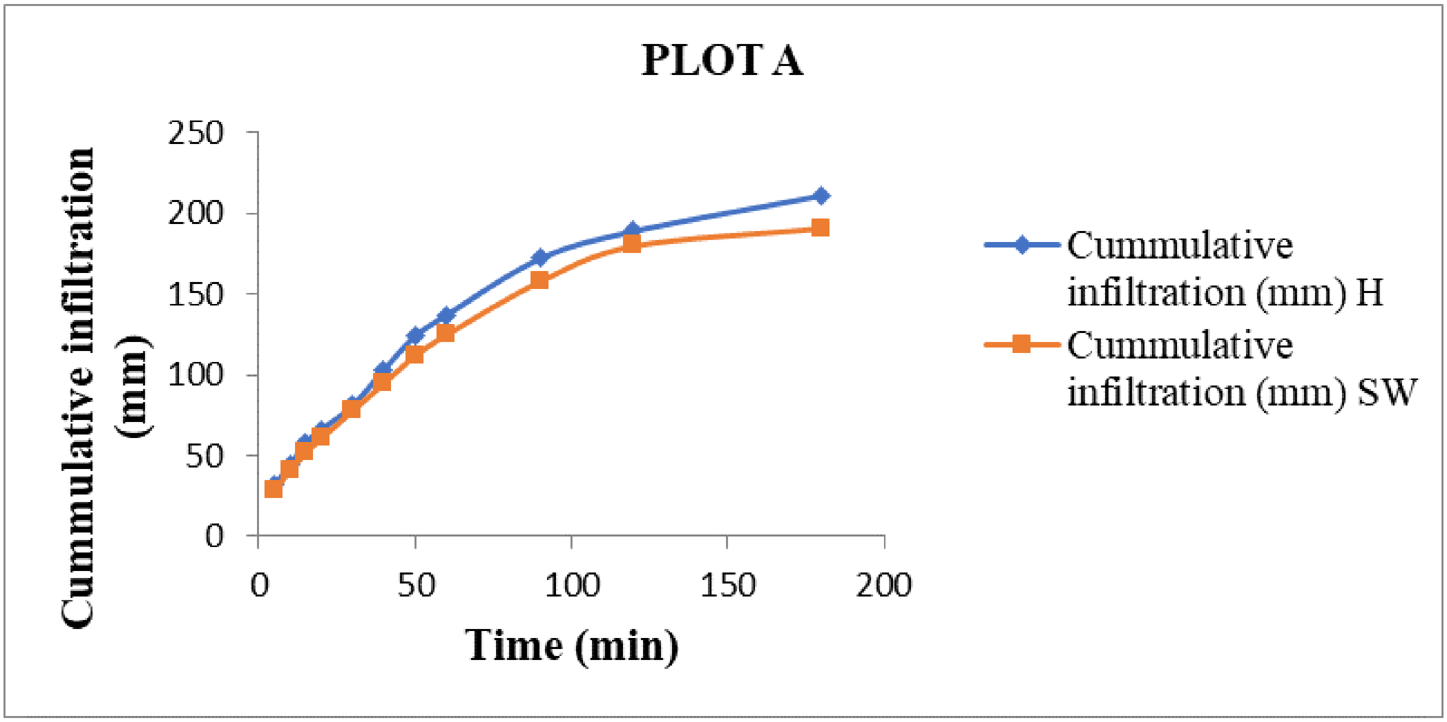

Fig 1: Average cumulative infiltration for Swartzendruber (SW) and Horton's (H) model for plot A

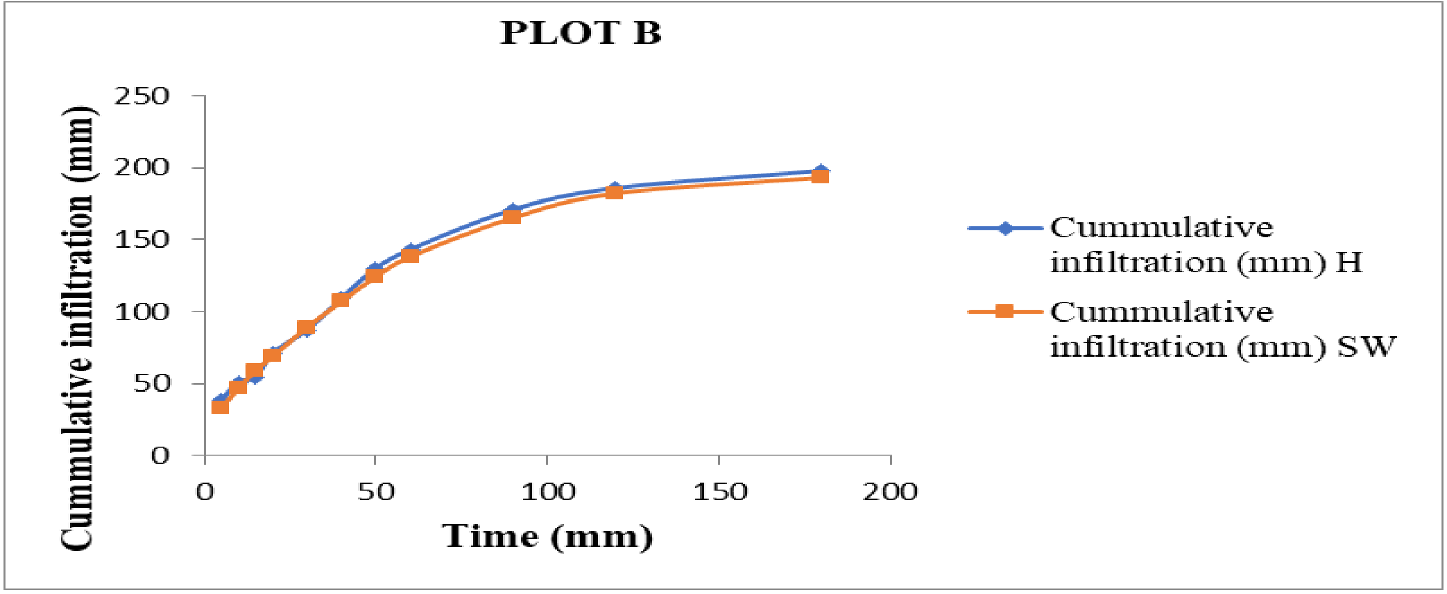

Fig 2: Average cumulative infiltration for Swartzendruber (SW) and Horton's (H) model for plot B

### *3.3. Models evaluation*

Table 4 shows the results for statistical parameters obtained from the two models evaluated. The performance indicators include the coefficient of determination ($R^2$), root mean square error (RMSE) and T-test statistics. The value of $R^2$ and RMSE recorded for Horton's model in plot A was 0.996 and 3.05, while 0.994 and 3.09 was recorded in plot B. The value of $R^2$ and RMSE recorded for Swartzendruber's model in plot A was 0.998 and 3.01, while 0.994 and 3.08 was recorded in plot B, respectively. However, the results of the evaluation indicated that both models were considered suitable due to the fact that they presented high values of $R^2$ and low values of RMSE. According to the T-test statistics, the values obtained for Horton's model in plot A and B were 2.93 and 2.51, while values obtained for Swartzendruber's model were 2.43 and 2.51 in plot A and B. These values indicated that there is a significant difference between the measured infiltrated depths from the field and predicted depths from the models because the calculated values were greater than the tabulated value of 2.365 at 5% confidence interval. This is consistent with findings of Aminu *et al*. (2019). However, the Swartzendruber's model was considered the best fit model because it presented the least calculated t value of 2.43.

Table 4: Statistical parameters for model evaluation from inflow-outflow data

| **Model** | **Parameters** | **PLOT A** | **PLOT B** |
|---|---|---|---|
| H | $R^2$ | 0.996 | 0.994 |
| | RMSE | 3.05 | 3.09 |
| | $\bar{d}$ | 18.12 | 16.8 |
| | $Sd$ | 17.49 | 19.10 |
| | $Se$ | 6.18 | 6.75 |
| | t | 2.93* | 2.51* |
| SW | $R^2$ | 0.998 | 0.994 |
| | RMSE | 3.01 | 3.09 |
| | $\bar{d}$ | 11.62 | 14.75 |
| | $Sd$ | 13.94 | 16.84 |
| | $Se$ | 4.92 | 5.95 |
| | t | 2.43* | 2.51* |

Tabulated value of t (5%) = 2.365;  = mean error,  = standard deviation and )= standard mean error, * = Significant

## 4.0. Conclusion

The models were successfully varied under field conditions to predict the infiltration rates of soils in the study area. In both cases (Swartzendruber and Horton's), the equation holds for rainfall and for furrow infiltration measurement. However, the study recommends that the two equations should be used in furrow irrigation measurements to predict infiltration depths of water in the study area.

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